



LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – NOVEMBER 2024

UPH 3502 – MATHEMATICAL PHYSICS - II



Date: 12-11-2024

Dept. No.

Max. : 100 Marks

Time: 09:00 am-12:00 pm

SECTION A - K1 (CO1)

Answer ALL the Questions

(10 x 1 = 10)

1. MCQ

a) Which of the following equation is an one-dimensional wave equation?

- (i) $\frac{\partial^2 y}{\partial x^2} = a \frac{\partial^2 y}{\partial t^2}$ (ii) $\frac{\partial y}{\partial x} = a \frac{\partial^2 y}{\partial t^2}$ (iii) $\frac{\partial y}{\partial x} = a^2 \frac{\partial^2 y}{\partial x^2}$ (iv) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$

b) In two-dimensional heat flow equation $\frac{\partial^2 u}{\partial t^2} = c^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]$, the term c^2 refers

- (i) thermal conductivity (ii) Specific heat
(iii) diffusivity (iv) heat flow from lower to higher temperature.

c) Fourier sin transform of $\frac{1}{x}$ is

- (i) $\frac{\pi}{2}$ (ii) $\frac{\sqrt{\pi}}{2}$ (iii) $\sqrt{\frac{\pi}{2}}$ (iv) $\sqrt{\frac{2}{\pi}}$

d) In Newton's forward interpolation formula the value of a_2 is given by

- (i) $a_2 = \frac{1}{h^2} \nabla y_0$ (ii) $a_2 = \frac{1}{h} \nabla y_0$ (iii) $a_2 = \frac{1}{2!h^2} \nabla^2 y_0$ (iv) $a_2 = \frac{1}{h^2} \nabla^2 y_0$

e) Which of the following is correct for Trapezoidal rule?

- (i) h ✗
(ii) $\frac{h}{2}$ ✗
(iii) $\frac{h}{4}$ ✗
(iv) $\frac{h}{3}$ ✗

2. Fill in the blanks

a) Laplace equation in cylindrical co-ordinate system is

b) If the roots α and β of second order differential equation are real and distinct then the general solution is

c) Fourier sine transform is defined as -----.

d) The word extrapolation is used to denote the process of finding the values of -----.

e) Numerical integration process when applied to a function of a single variable, then it is known as -----.

SECTION A - K2 (CO1)

Answer ALL the Questions

(10 x 1 = 10)

3. True or False

a)	Partial differential equation can be formed either by eliminating arbitrary constants or functions
b)	The equation of heat-flow in polar coordinates for steady-state is $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = \frac{1}{\alpha^2} \frac{\partial u}{\partial t}$.
c)	The convolution of two function is defined as $f(x) * g(x) = \int_{-\infty}^{\infty} f(u)g(x-u)du$.
d)	The Newton's forward interpolation formula contain the term y_0 .and the forward difference of y_0
e)	In Euler's method, the actual curve is approximated by a sequence of long straight line.

4.	Match the following
a)	Assumption in method of separation : for unequally spaced value of x
b)	$F_s[e^{-ax}] : \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$
c)	$F[af_1(x) + bf_2(x)] : u_x = \frac{\partial u}{\partial x}, u_y = \frac{\partial u}{\partial y}$
d)	Lagrange's interpolation formula : $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{isx} [af_1(x) + bf_2(x)] dx$
e)	Simpson's one-third rule : $\sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2}$

SECTION B - K3 (CO2)

Answer any TWO of the following.

(2 x 10 = 20)

5.	By method of separation of variables solve the partial differential equation $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$
6.	Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$
7.	Using the method of least squares, fit the curve $y = ax^2 + \frac{b}{x}$ to the following data X: 1 2 3 4 Y: -1.51 0.99 3.88 7.66
8.	Evaluate the integral $\int_0^1 \frac{x^2}{1+x^3} dx$ using Simpson's 1/3rd rule. Compare the error with exact value.

SECTION C – K4 (CO3)

Answer any TWO of the following .

(2 x 10 = 20)

9.	Solve $\frac{\partial^2 z}{\partial x^2} + 2 \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$
10.	Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary condition $u(x,0) = 3 \sin n\pi x$. $u(0,t), u(l,t) = 0$ where $0 < x < l$.
11.	Apply Lagrange's formula to calculate f(3) from the following data: X: 0 1 2 5 Y: 2 3 12 147
12.	Using improved Euler method find y at x = 0, 0.1 and 0.2 given $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$.

SECTION D – K5 (CO4)

Answer any ONE of the following

(1 x 20 = 20)

13.	Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in a rectangle in the xy -plane with $u(x,0)=0, u(x,b)=0, u(0,y)$ and $u(a,y)=f(y)$ parallel to y axis
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14.	Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method.					
SECTION E – K6 (CO5)						
Answer any ONE of the following				(1 x 20 = 20)		
15.	Find Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$.					
16.	The population of certain town is given below. Find the rate of growth of the population in 1931, 1941, 1951, 1961 and 1971.					
	Year X:	1931	1941	1951	1961	1971
	Population in thousand Y:	40.62	60.80	79.95	103.56	132.65

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